

Math 16B: Homework 5

Due: July 30

- Find the n th Taylor polynomial for each of the following functions at the given point:
 - $n = 2$: $f(x) = x^{1/3}$ at $x = 27$.
 - $n = 2$: $f(x) = \tan(x)$ at $x = \frac{\pi}{4}$.
 - $n = 3$: $f(x) = \frac{1}{7-x}$ at $x = 5$.
 - $n = 4$: $f(x) = \ln(x)$ at $x = 1$.
- For each of the following problems, write a differential equation to describe the physical system, solve it and use the solution to address the queries.
 - The population of a town grows at a rate proportional to the population present at time t . The initial population of 500 increases by 15% in 10 years. What will be the population in 30 years?
 - A thermometer reading 70° F is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer reads 110° F after $\frac{1}{2}$ minute and 145° F after 1 minute. Assuming that the heating process obeys Newton's Law of Heating, how hot is the oven?
 - A dead body was found within a closed room of a house where the temperature was a constant 70° F. At the time of discovery the core temperature of the body was determined to be 85° F. One hour later a second measurement showed that the core temperature of the body was 80° F. Assume that the core temperature of the body at the beginning was 98.6° F. Determine how many hours elapsed before the body was found.
 - A large tank is partially filled with 100 gallons of fluid in which 10 pounds of salt is dissolved. A highly concentrated salt solution containing $\frac{1}{2}$ pound of salt per gallon is pumped into the tank at a rate of 6 gal/min. The well-mixed solution is then pumped out at a slower rate of 4 gal/min. Find the number of pounds of salt in the tank after 30 minutes.
 - A vat with 500 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after an hour?
- Sketch the solutions of the following differential equations for the given initial conditions. Also include the constant solutions:

- (a) $y' = -y(y - 3)$ with initial conditions $y(0) = -4$, $y(0) = 1$ and $y(0) = 2$.
- (b) $y' = \cos(y)$ with initial conditions $y(0) = -\frac{\pi}{3}$, $y(0) = \frac{\pi}{3}$ and $y(0) = \frac{5\pi}{3}$.
- (c) $y' = y^3 - y$ with initial conditions $y(0) = -3$, $y(0) = -0.5$ and $y(0) = 2$.

4. For each of the following problems, write a differential equation to describe the physical system and sketch its appropriate solutions to address the queries.

- (a) The number of students boarding in a dorm is 200. At the beginning of a new term, suppose one student returns with a mild infection. Assume that the rate at which the number of infected students grows is proportional to the product of the infected number and the uninfected number. Assuming that no one uses any medication, argue that at least 199 of the students (including the original infected) will catch the infection.
- (b) A savings account needs to be set up that pays \$800 steadily per year. Suppose that the bank compounds continuously at 4%. What is the least initial amount that should be invested to ensure that the payments continue indefinitely?